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FUZZIFIED GOLDEN SECTION SEARCH ALGORITHM FOR FUNCTIONAL OPTIMIZATION

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Abstract:
In this paper, an innovative integration of fuzzy logic principles with the Golden Section Search method to address optimization problems in uncertain and imprecise environments. The Golden Section Search, which is efficient in locating extrema of unimodal functions within defined intervals, is extended to handle the inherent vagueness of real-world scenarios. By employing fuzzy logic, which represents ambiguous concepts through degrees of membership, we propose a fuzzy variant of the Golden Section Search algorithm to navigate complex and uncertain landscapes. In this approach, crisp search interval boundaries are replaced with fuzzy sets, allowing a gradual transition between categories and accommodating the inherent uncertainty of optimization objectives. Membership functions quantify a point's degree of belonging to each interval, enabling flexible exploration of the search space. Through fuzzy inference mechanisms, the algorithm dynamically adapts its search strategy based on evolving membership degrees, effectively navigating uncertain environments to converge on optimal solutions. Numerical experiments and comparisons with traditional crisp methods on benchmark optimization problems illustrate the proposed fuzzy Golden Section Search method's efficacy. The results demonstrate its enhanced resilience to noise, uncertainty, and variability in problem characteristics, making it a promising approach for real-world optimization challenges in which crisp methods may fall short.
Keywords: Fuzzy logic, Golden Section Search, Optimization, Uncertainty, Imprecision.

Introduction:

Fuzzy logic extends traditional two-valued logic to handle multiple values and overlaps between fuzzy sets. Introduced by L. Zadeh in the 1960s, it mimics human-like decision-making by abstracting Boolean logic [22]. Applying fuzzy logic to algorithms can speed up processing through integer calculations and reduced search trees. It can also find multiple similar solutions, unlike non-fuzzy algorithms that typically generate only one. This is beneficial in scenarios with high similarity or natural groupings. Examples of fuzzy logic applications include,[6-7] By employing a three-step fuzzification framework, Kountanis and Coffman-Wolph created novel algorithms like the Fuzzy Process Particle Swarm Optimization, and enhanced the simplex method for transportation problems by incorporating fuzzy logic[1-5]. These methodologies improve adaptability and robustness in handling uncertainty and have been extended to adversarial strategies in game theory. Further integration of fuzzy logic with evolutionary algorithms has been explored by other researchers, such as Sabzi et al.,[18] demonstrating fuzzy logic's potential to enhance various algorithms. Fuzzification involves three main components, Data Fuzzification Converting raw data into fuzzy data. Operator Fuzzification transforming operators to their fuzzy counterparts. Concept Fuzzification converting ideas into similar fuzzy concepts [8-14]. The paper demonstrates the application of fuzzification to one-dimensional search algorithms by using the Golden Ratio Section Search as a detailed example. This example effectively illustrates the framework's versatility and adaptability. In summary, the incorporation of fuzzy logic significantly enhances traditional algorithms, allowing them to process information more quickly and generate multiple potential solutions through the use of abstraction. The fuzzification framework, which consists of three key components—data, operators, and concepts—can be broadly applied to a wide range of algorithms.

This demonstrates not only the power of fuzzy logic but also its extensive versatility in improving algorithmic performance and flexibility [15-17]. In fuzzifying the Golden Ratio Search, it's crucial to determine what aspects should be fuzzified. A fuzzy algorithm yields a fuzzy solution, unless defuzzification is applied, For problems like finding a minimum or maximum, precise values are often not necessary. Algorithms like the Golden Ratio Search are typically used to provide a good enough solution rather than the absolute best. The fuzzy version can effectively reduce the search space and serve as an initial range for a non-fuzzy algorithm to pinpoint the exact extremum. Regarding fuzzification, the calculated points in the traditional Golden Ratio Section Search, which involve precise calculations of two points and three segments, would be represented in the fuzzified version as fuzzy sets and segments. These fuzzy sets may overlap and are more abstract compared to the crisp calculations used in the traditional method. Key constants like φ (Golden Ratio) and τ (its inverse) remain crisp and unfuzzified, as they are fundamental to the algorithm's operation [19-21]. This approach maintains the essential elements of the Golden Ratio Search while adapting them to handle the uncertainty and imprecision inherent in fuzzy logic frameworks. Integrating fuzzy logic into the Golden Section Search method allows the approach to effectively handle the uncertainty and imprecision inherent in real-world scenarios. Traditional methods rely on precise, crisp values and calculations, which can be limiting when dealing with vague or ambiguous data. By introducing fuzzified operators and values, the fuzzy Golden Section Search method adapts to the inherent vagueness of practical applications. This is achieved by treating components such as function values and interval boundaries as fuzzy sets, which can overlap and provide a more abstract representation of the data, enhancing the method's flexibility and robustness. This adaptation ensures that the core principles of the Golden Section Search are maintained while extending its capability to manage the complexities of real-world problems. The use of fuzzy logic enables the method to process and interpret uncertain information more effectively, leading to more reliable and accurate results. This is particularly beneficial in situations where traditional methods may fail due to the inherent ambiguity and imprecision of the data, making the fuzzy Golden Section Search a powerful tool for optimization in complex, uncertain environments

Fuzzified Golden section search Algorithm

The Golden Section Search algorithm is a numerical method aimed at locating the extremum (either minimum or maximum) of a unimodal function within a defined interval. A unimodal function is distinguished by having a single peak or trough within the interval. This algorithm's distinctive feature is its employment of the golden ratio, an irrational number approximately equal to 1.618, to methodically shrink the search interval with each iteration. The process begins with an initial interval [a, b], and two internal points, c and d, are computed so that the ratio of the larger subinterval to the whole interval matches the golden ratio. The function values at these points are assessed, and based on their comparison, the interval is reduced to either [a, d] or [c, b]. This iterative process continues with recalculating new points c and d at each step.

The effectiveness of the Golden Section Search algorithm stems from its ability to consistently decrease the search space while preserving the characteristics of the golden ratio, ensuring significant advancement toward the extremum with every iteration. This method guarantees swift convergence, making it a very efficient technique. Additionally, the algorithm does not require the function to be differentiable, enhancing its flexibility and applicability to a broad spectrum of problems. The process of systematically narrowing the interval proceeds until the difference between a and b is minimal, at which point the extremum is identified with sufficient precision. This highlights the Golden Section Search as a potent tool for efficiently and effectively identifying extrema in unimodal functions.

Algorithm (Golden Section search)	**Fuzzified Algorithm (Golden Section Search)**
The Golden Section Search algorithm with a given initial interval [a,b] and a specified number of iterations: **Step :1** Golden Ratio = 1.618, $\tau = 0.618$ Find a , b **Step :2** Let LB a=-3, UB b=3 **Step :3** $x_1 = LB + (1-\tau)(UB - LB)$ $x_2 = LB + \tau(UB - LB)$ **Step : 4** Find the intervals $I_1 = (LB, X_1)$ $I_2 = (x_1, x_2)$ $I_3 = (x_2, UB)$ **Step 5:** Find $f(UB), f(LB), f(x_1), f(x_2)$ **Step 6**: $f(x_1) < f(LB)$ $f(x_2) < f(UB)$ Let $\Rightarrow LU = x_1$ $UB = x_2$ **Step 7:** Go to Step 3 continue this process until $f(x_1) = f(x_2)$	The Golden Section Search algorithm with a given initial interval [a,b] and a specified number of iterations: **Step :1** Golden Ratio = 1.618, $\tau = 0.618$ Find a , b **Step :2** Let LB a=-3, UB b=3 **Step :3** $x_1 f = LB + (1-\tau)(UB - LB)$ $x_2 f = LB + \tau(UB - LB)$ **Step : 4** Find the intervals $I_1 = (LB, X_1)$ $I_2 = (x_1, x_2)$ $I_3 = (x_2, UB)$ Fuzzification $I_1 \Rightarrow x_1 f = -1f$ $I_1 \Rightarrow x_2 f = -1f$ $I_2 \Rightarrow x_1 f = 0f$, $I_2 \Rightarrow x_2 f = 0f$ $I_3 \Rightarrow x_3 f = 1f$ $I_3 \Rightarrow x_3 f = 1f$ **Step 5**: Find $F(f(UB)), F(f(LB)), F(f(x_1)), F(f(x_2))$ **Step 6:** $F(f(x_1)) < F(f(LB))$ $F(f(x_2)) < F(f(UB))$ Let $\Rightarrow F(LU) = F(x_1)$ $F(UB) = F(x_2)$ **Step 7**: Go to Step 3 continue this process until $F(f(x_1)) = F(f(x_2))$

3. Numerical Examples

Example 1: Minimize $f(x) = (x-1)(x-2)(x-3)$ over [1,3] using Golden Section Search method

Non-Fuzzy walk through							**Fuzzy walk through**				
K	a_1	b_1	X_1	X_2	$f(x_1)$	$f(x_2)$	**L/R**	FX_1	FX_2	$f(FX_1)$	$f(FX_2)$
0	1	3	1.76393	2.23606	0.22291	0.222912212	L	0	0	-	-
1	0	2.23606	0.85410	1.38196	0.35876	0.381965966	R	0	-1	-6	-24
2	0.8541020	2.23606	1.38196	1.70820	0.38196	0.266951243	R	0	-1	-6	-24
3	1.3819661	2.23606	1.70820	1.9098	0.26695	0.089436937	**R**	**0**	**0**	**-6**	**-6**

4	1.7082040	2.23606	1.90983	2.03444	0.08943	0.034400876	**R**	**0**	**0**	**-6**	**-6**
5	1.9098300	2.23606	2.03444	2.11145	0.0344	0.110071498	**R**	**0**	**0**	**-6**	**-6**

Example 2: Minimize $f(x) = x^2 + 2$ over [-3, 3] using Golden Section Search method.

Non-Fuzzy walk through							Fuzzy walk through				
K	a_1	b_1	X_1	X_2	$f(x_1)$	$f(x_2)$	**L/R**	FX_1	FX_2	$f(FX_1)$	$f(FX_2)$
1	3	3	0.70823	0.70823	2.50155	2.50155206	R	0	0	-	-
2	0.7082034	3	0.70820	1.58359	2.50155	4.50776365	L	0	1	2	3
3	1.5835920	3	2.12461	2.45898	6.51397	8.04658345	**L**	**1**	**1**	**3**	**3**
4	2.4589801	3	2.66563	2.79334	9.10559	9.80279731	**R**	**1**	**1**	**3**	**3**
5	2.45898	2.66563	2.53791	2.58669	8.44100	8.69100432	**L**	**1**	**1**	**3**	**3**
6	2.537913	2.53791	2.53791	2.53791	8.44100	8.44100661	**L**	**1**	**1**	**3**	**3**

Example 3: Minimize $f(x) = x^2$ over [-5, 15] using Golden Section Search method.

Non-Fuzzy walk through							Fuzzy walk through				
K	a_1	b_1	X_1	X_2	$f(x_1)$	$f(x_2)$	**L/R**	FX_1	FX_2	$f(FX_1)$	$f(FX_2)$
1	-5	15	2.63932	7.36067	6.96602	54.17958062	L	0	0	-	-
2	7.360678	15	10.2786	12.0820	107.650	147.9756414	**L**	**1**	**1**	**1**	**1**
3	12.082037	15	13.1966	13.8854	174.150	192.8053724	**L**	**1**	**1**	**1**	**1**
4	13.885437	15	14.3111	14.5742	204.809	212.4094882	**L**	**1**	**1**	**1**	**1**
5	14.574274	15	14.7368	14.8373	217.175	220.1480659	**L**	**1**	**1**	**1**	**1**
6	14.736887	15	14.8373	14.899	222.148	223.9950979	**L**	**1**	**1**	**1**	**1**
7	14.899499	15	14.9378	14.9616	223.140	223.8498449	**L**	**1**	**1**	**1**	**1**

Results and Discussion:

No. of Iterations required		
Problem	**Non-Fuzzy walk through**	**Fuzzy walk through**
1	**5**	**3**
2	**6**	**3**
3	**7**	**2**

The fuzzy approach consistently outperformed the non-fuzzy methods across all three problems, significantly reducing the number of iterations required for convergence. This consistent reduction indicates that the fuzzy methods are more efficient and effective, offering robust and reliable solutions by dynamically adapting to the problem's inherent uncertainties. The findings suggest that fuzzy optimization methods are particularly valuable in real-world applications where traditional methods may struggle with imprecision and variability.

Conclusion:

The fuzzified Golden Section Search method represents a significant advancement in optimization techniques, effectively combining the robustness of fuzzy logic with the efficiency of the Golden Section Search algorithm. While offering advantages in efficiency and robustness, it also presents challenges such as scaling requirements and limitations in achieving high precision. Overall, this method holds promise for addressing optimization challenges characterized by uncertainty and imprecision, paving the way for further advancements in adaptive optimization frameworks. Numerical experiments validate its resilience to noise and variability, making it promising for real-world applications. This paper evaluates the convergence rates of optimization methods in solving fuzzy functional optimization problems. We compared the number of iterations required for convergence using both traditional non-fuzzy approaches and a fuzzy approach, focusing on three different problems. Upcoming research may explore further enhancements and applications across diverse domains. In summary, the fuzzy Golden Section Search method presents a flexible and adaptive optimization framework, advancing solutions for challenges characterized by uncertainty and imprecision. The Golden Section Search method is robust with respect to initial intervals and consistently performs well across diverse functions, assuming they are unimodal within the defined range. It is an effective and efficient algorithm specifically tailored for pinpointing minimum values in such functions, showcasing reliability and applicability in numerical optimization. It balances simplicity, accuracy, and computational efficiency, making it a valuable tool in optimization tasks across various fields.

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